Waveguide Composed of Pinhole Array

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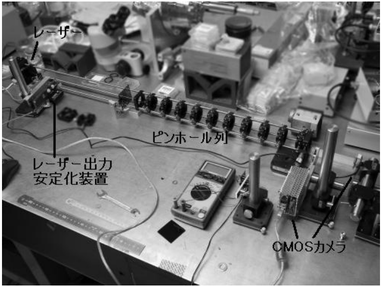
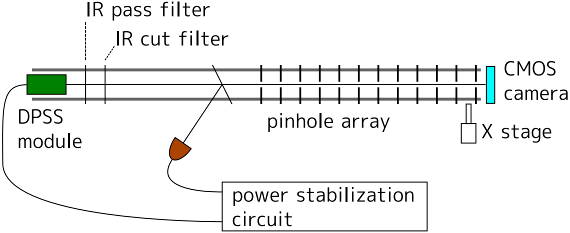
abstract

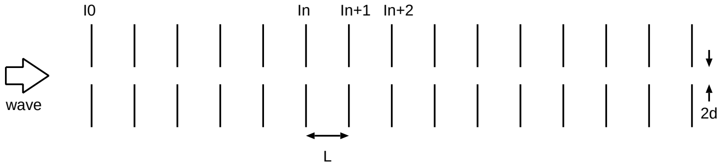
A waveguide composed of an array of pinholes is proposed. Intensity loss at each pinhole is shown to be proportional to $\mathbf{L}^{1.5}$ where $\mathbf{L}$ is the distance between adjascent pinholes. Thus by increasing the number of pinholes 4 times, the loss for unit length becomes a half. Experimental demonstration of such waveguide is also given.

guiding of wave (light, atoms,...) with pinholes/slits





λ =1064nm, 532nm



$$\mathbf{I_{n+1}} = \left\{ \mathbf{1} - \beta \left(\frac{\lambda \mathbf{L}}{\mathbf{d}^2} \right)^{\frac{3}{2}} \right\} \mathbf{I_n}$$

By reducing the pinhole/slit spacing 1/4 the loss par unit length becomes a half!

Propagation through the pinhole array

Propagation process can be devided into two parts:

1. Free propagation between two neighboring slits

$$\mathbf{T_L} = \int_{-\infty}^{\infty} \mathbf{dk}\ |\mathbf{k}\rangle \mathbf{e}^{\mathbf{i k_z L}} \langle \mathbf{k}| = \mathbf{e}^{\mathbf{i}\phi_0} \int_{-\infty}^{\infty} \mathbf{dk}\ |\mathbf{k}\rangle \mathbf{e}^{-\mathbf{i}\alpha \mathbf{k}^2} \langle \mathbf{k}|$$

$$\mathbf{k_0^2} = \mathbf{k_z^2} + \mathbf{k^2} \rightarrow \mathbf{k_z} \sim \mathbf{k_0} - \frac{\mathbf{k^2}}{\mathbf{2k_0}} \rightarrow \alpha = \frac{\mathbf{L}}{\mathbf{2k_0}}$$

2. Masking of wavefunction when the wave pass through a slit

$$(\mathbf{T_M}\psi)(\mathbf{x}) = \begin{cases} \psi(\mathbf{x}) & |\mathbf{x}| < \mathbf{d} \\ \mathbf{0} & \textbf{otherwise} \end{cases}$$

3. $\mathbf{T} \equiv \mathbf{T_M}\, \mathbf{T_L}$

4. Propagation through many slits: $\mathbf{T^n}$

5. Find eigenvalue and eigenstate of $\mathbf{T}$

Wavefunction just after a slit

- Takes nonzero value only inside the opening
- Can be expanded with sin/cos function that are 0 on the slit boundary:

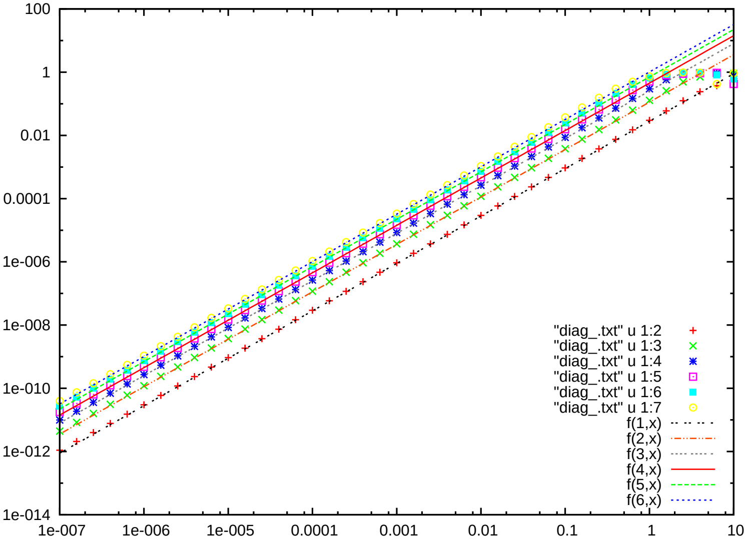
$$\psi_n(x) = \begin{cases} c_0 \cos k_n x & (n : \text{even}) \\ c_0 \sin k_n x & (n : \text{odd}) \\ 0 & \end{cases} \quad \begin{matrix} (-d \leq x \leq d) \\ \\ (\text{otherwise}) \end{matrix}$$

$k_n = \frac{n+1}{2d}\pi \ (n = 0, 1, 2, \ldots)$

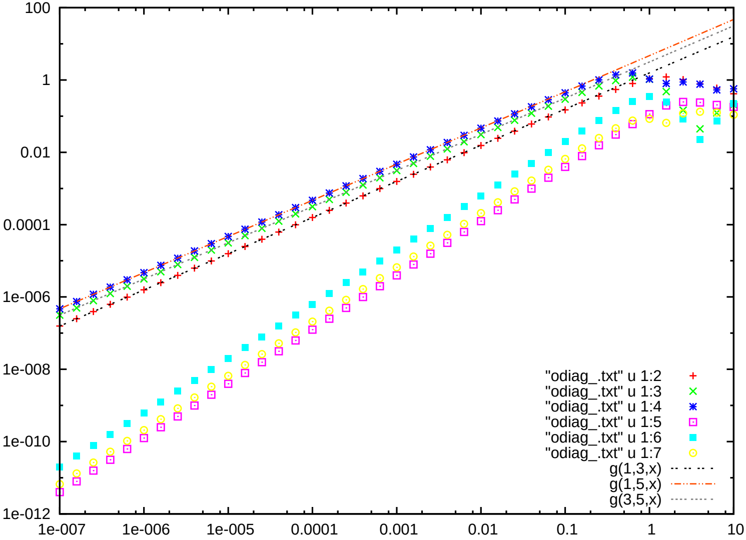
$c_0 = \frac{1}{\sqrt{d}}$: normalization constant

(ψ_n are eigenfunctions of "Particle in a Box")

$1 - |T_{nn}|$


100
1
0.01
0.0001
1e-006
1e-008
1e-010
1e-012
1e-014
1e-007
1e-006
1e-005
0.0001
0.001
0.01
0.1
1
10
"diag_.txt" u 1:2
"diag_.txt" u 1:3
"diag_.txt" u 1:4
"diag_.txt" u 1:5
"diag_.txt" u 1:6
"diag_.txt" u 1:7
f(1,x)
f(2,x)
f(3,x)
f(4,x)
f(5,x)
f(6,x)

off diagonal elements v.s. difference of the diagonal elements of **T**



asymptotic form of the operator **T**

$$\mathbf{T_{mn}} = \delta_{mn} \exp\left(-i\frac{(m+1)^2\pi^2}{8}\frac{L}{kd^2}\right) + \frac{1+(-1)^{m+n}}{2}(m+1)(n+1)\frac{\pi^{\frac{3}{2}}}{12}\left(\frac{L}{kd^2}\right)^{\frac{3}{2}}(-1-i) + O\left(\left(\frac{L}{kd^2}\right)^2\right)$$

amplitude loss par single unit of slit (length **L**):

$$1 - |T_{mm}| = (m+1)^2\frac{\pi^{\frac{3}{2}}}{12}\left(\frac{L}{kd^2}\right)^{\frac{3}{2}} + O\left(\left(\frac{L}{kd^2}\right)^2\right)$$

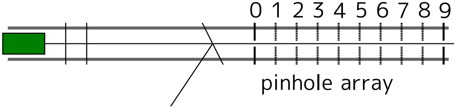
eigenstates & eigenvalues of $\mathbf{T}$

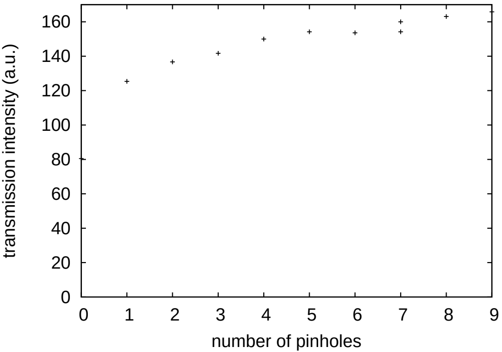
- difference of the diagonal elements: $\mathbf{O}\left(\frac{\lambda L}{d^2}\right)$
- off-diagonal elements: $\mathbf{O}\left(\frac{\lambda L}{d^2}\right)^{1.5}$

When $\frac{\lambda L}{d^2} \to 0$

- ψ_n become eigenstates
- $\mathbf{T}_{nn}$ become eigenvalues

light power after the pinhole no.9

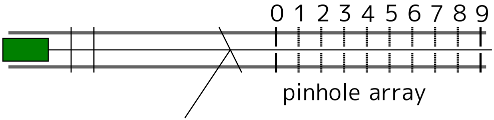




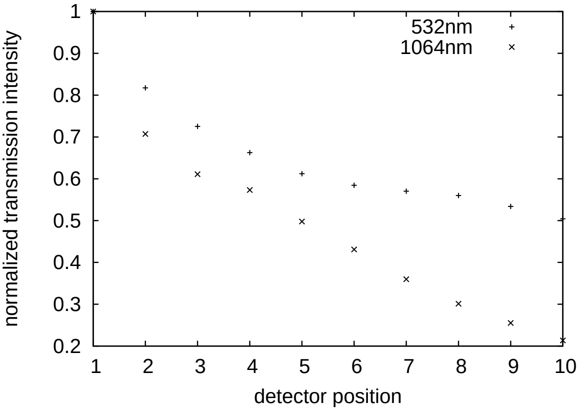
$\lambda = \mathbf{532}$nm
$\phi = \mathbf{0.5}$mm
$\mathbf{L} = \mathbf{45}$mm

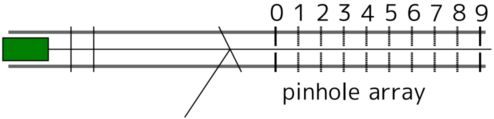
0: pinhole no.9 only,
1: pinhole no.9 + no.0,
2: pinhole no.9 + no.0 + no.1,
...

160 in the vertical axis corresponds to about 50% of light power transmitted through pinhole no.0.

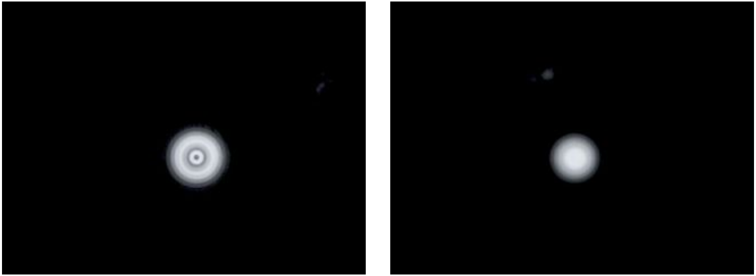


light power after each pinhole



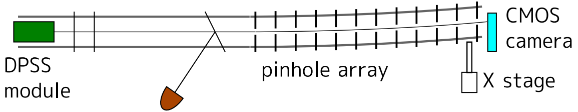


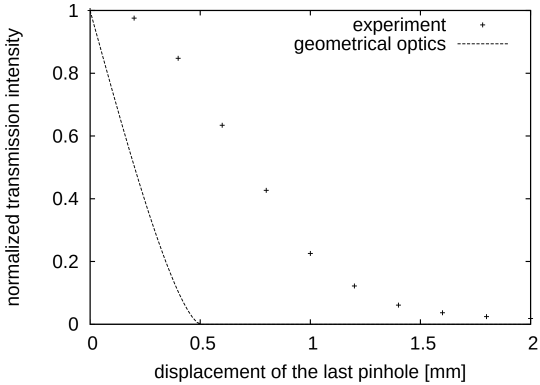
beam profile after pinhole no.0 and no.3



$\lambda = 532\text{nm},\ \phi = 0.5\text{mm},\ \mathsf{L} = 45\text{mm}$

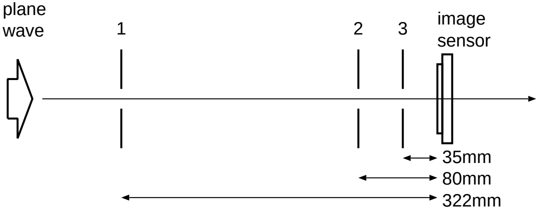
beam deflection





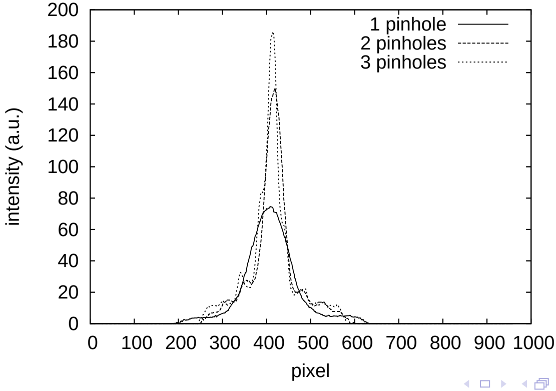
$$\lambda = 532\text{nm},\ \phi = 0.5\text{mm},\ L = 45\text{mm}$$

beam focusing



λ=1064nm

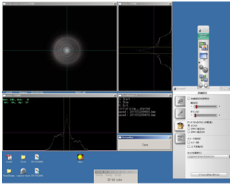
ϕ=1.0mm



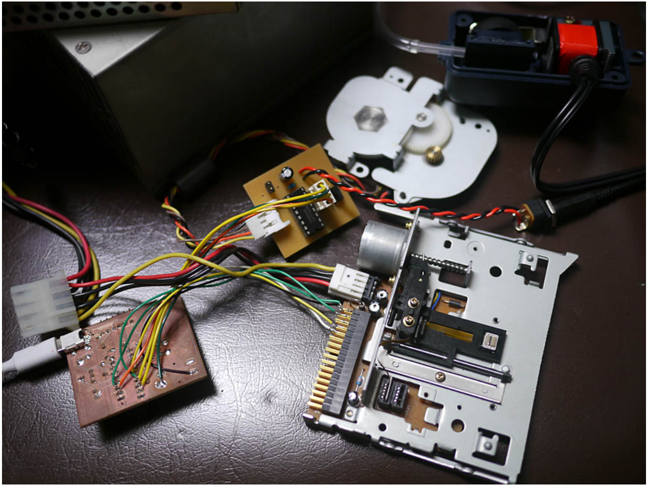
features of this methods application

- Any kind of wave can be handled
  - electro-magnetic wave of special wavelength, matter wave, etc.
- possible to bend
- wave travels through a free space
  - guide atoms by the light guided by the pinhole array
- manipulation of the transverse mode

- FREE beam profiling program for WDM cameras (USB cameras) is available at: http://m.ils.uec.ac.jp/sbpw/



pinhole aligning ROBOT (under construction)



number of pinholes: $\sim$ **100**

pinhole spacing: $\gtrsim$ **2**mm

naive estimate of the loss coefficient

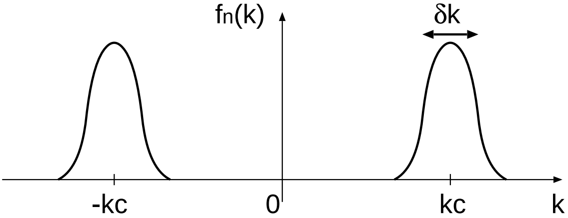
- projection of $\mathbf{T}\psi$ onto ψ

$$\mathbf{S} = \langle\psi|\mathbf{T}|\psi\rangle = \langle\psi|\mathbf{T_L}|\psi\rangle = \int \mathbf{dk}\ |\langle\psi|\mathbf{k}\rangle|^2 \mathbf{e}^{-\mathbf{i}\alpha\mathbf{k}^2}$$

- $\mathbf{1} - |\mathbf{S_n}|^2 = \mathbf{1} - |\langle\psi_\mathbf{n}|\mathbf{T}|\psi_\mathbf{n}\rangle|^2$ gives the intensity loss coefficient for the $\mathbf{n}$th mode when $\frac{\lambda \mathbf{L}}{\mathbf{d}^2} \ll \mathbf{1}$
- property of $\mathbf{f(k)} \equiv |\langle\psi|\mathbf{k}\rangle|^2$:

$$\int_{-\infty}^{\infty} \mathbf{dk}\ \mathbf{f(k)} = \mathbf{1}, \quad \mathbf{f(k)} \geq \mathbf{0}$$

$$\mathbf{f_n(k) \equiv |\langle \psi_n | k \rangle|^2}$$



$\mathbf{k_c = \frac{n+1}{2d}\pi}$、 $\boldsymbol{\delta k \sim \frac{\pi}{2d}}$。

$$\mathbf{k_c \equiv \int_0^\infty dk\ k\{f(k) + f(-k)\}}$$

$$\boldsymbol{\delta k \equiv \left(\int_0^\infty dk\ (k - k_c)^2\{f(k) + f(-k)\}\right)^{\frac{1}{2}}}$$

$$
\begin{aligned}
& \mathbf{S} \\
= \ & \langle\psi|\mathbf{T}|\psi\rangle \\
= \ & \int_{-\infty}^{\infty} dk\ f(k)e^{-i\alpha k^2} \\
= \ & e^{-i\alpha k_c^2}\int_0^{\infty} dk\ \{f(k)+f(-k)\}e^{-i\alpha(2k_c\Delta k+\Delta k^2)} \\
\sim \ & e^{-i\alpha k_c^2}\int_0^{\infty} dk\ g(k)\{1-i\alpha(2k_c\Delta k+\Delta k^2)-2\alpha^2k_c^2\Delta k^2\} \\
= \ & e^{-i\alpha k_c^2}\{1+(-i\alpha-2\alpha^2k_c^2)\delta k^2\}
\end{aligned}
$$

$$|S|^2 \sim 1-4\alpha^2k_c^2\delta k^2 = 1-\frac{k_c^2\delta k^2}{k_0^2}L^2$$

$$|S_n|^2 \sim 1-\frac{(n+1)^2\pi^4}{16k_0^2d^4}L^2 = 1-\frac{(n+1)^2\pi^2}{64}\left(\frac{\lambda L}{d^2}\right)^2$$

wrong result!

$g(k)=f(k)+f(-k)$, $\Delta k \equiv k-k_c$.