1 Second order photon correlation in the presence of LO photons

1.1 Conventions, etc.

$$E(t) = \int_{-\infty}^{\infty} E(\omega) e^{i\omega t} d\omega \tag{1}$$

$E^*(\omega) = E(-\omega)$ because $E(t)$ is real valued.

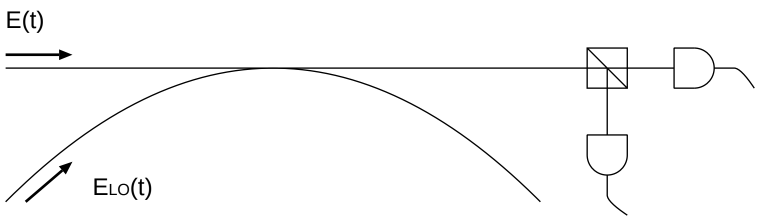
$$\begin{cases} E^{(+)}(t) \equiv & \int_0^{\infty} E(\omega) e^{i\omega t} d\omega \\ E^{(-)}(t) \equiv & \int_{-\infty}^{0} E(\omega) e^{i\omega t} d\omega \end{cases} \tag{2}$$

$E^{(+)}(t)$: positive frequency component.
$E^{(-)}(t)$: negative frequency component.

$E^{(+)}(t)^* = E^{(-)}(t)$.

$$E(t) = E^{(+)}(t) + E^{(-)}(t)$$

1.2 Beat signal



$E_{tot}(t) = E(t) + E_{LO}(t)$
$E_{LO}^{(+)}(t) = E_0 \exp(i\omega_{LO} t)$

$I = \langle E^{(-)}(t) E^{(+)}(t) \rangle$
$I_{LO} = |E_0|^2$

Two photon correlation in the presence of LO photons:

$$\langle E_{tot}^{(-)}(t) E_{tot}^{(-)}(t+\tau) E_{tot}^{(+)}(t+\tau) E_{tot}^{(+)}(t) \rangle = A_1 + A_2 + A_3 \tag{3}$$

where

$$A_1 = \langle E^{(-)}(t) E^{(-)}(t+\tau) E^{(+)}(t+\tau) E^{(+)}(t) \rangle \tag{4}$$

$$A_2 = I_{LO}\, g^{(1)}(\tau) \exp(-i\omega_{LO}\tau) + c.c. \tag{5}$$

$$A_3 = 2II_{LO} + I_{LO}^2 \tag{6}$$

with $g^{(1)}(\tau) \equiv \langle E^{(-)}(t)E^{(+)}(t+\tau)\rangle$.
If we Fourier transform the two photon correlation data, then we obtain the Fourier transform of $g^{(1)}(\tau)$ offset by $-\omega_{LO}$ (and not that of $|g^{(1)}(\tau)|$). *See also next section.*

1.3 Example 1

For a Lorentzian line shape of width δ centered at ω_0,

$$g^{(1)}(\tau) = e^{-\delta|\tau|}\exp(i\omega_0\tau) \tag{7}$$

and thus A_2 is reduced to

$$A_2 = 2I_{LO}\,|g^{(1)}(\tau)|\cos(\omega_0 - \omega_{LO})\tau \tag{8}$$

which coincides with the expression in the ref.[1].

1.4 Example 2

Suppose $E(t)$ has two frequency components ω_1 and ω_2 (with Lorentzian line width δ_1 and δ_2) with relative intensity α and β,

$$g^{(1)}(\tau) = \alpha e^{-\delta_1|\tau|}\exp(i\omega_1\tau) + \beta e^{-\delta_2|\tau|}\exp(i\omega_2\tau) \tag{9}$$

then

$$A_2 = 2I_{LO}\{\alpha e^{-\delta_1|\tau|}\cos(\omega_1 - \omega_{LO})\tau + \beta e^{-\delta_2|\tau|}\cos(\omega_2 - \omega_{LO})\tau\} \tag{10}$$

which cannot be written in the form of (8).

2 $g^{(1)}(\tau)$ and the spectrum

$$\begin{aligned} g^{(1)}(\tau) \equiv&\ \langle E^{(-)}(t)E^{(+)}(t+\tau)\rangle \\ =&\ \int_{-\infty}^{\infty}\left\{\int_0^{\infty}E^*(\omega)e^{-i\omega t}d\omega\int_0^{\infty}E(\omega')e^{i\omega'(t+\tau)}d\omega'\right\}dt \\ =&\ 2\pi\int_0^{\infty}|E(\omega)|^2e^{i\omega\tau}d\omega \end{aligned} \tag{11}$$

References

[1] Hyun-Gue Hong, Wontaek Seo, Moonjoo Lee, Wonshik Choi, Jai-Hyung Lee, and Kyungwon An, "Spectral line-shape measurement of an extremely weak amplitude-fluctuating light source by photon-counting-based second-order correlation spectroscopy," Opt. Lett. 31, 3182-3184 (2006)